RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. THIRD SEMESTER EXAMINATION, DECEMBER 2017

SECOND YEAR [BATCH 2016-19]

Date : 22/12/2017 **MATHEMATICS [General]**

Time : 11 am – 2 pm **Paper : III** Full Marks : 75

[Use a separate Answer Book for each Group]

Group – A

(*Answer any four questions*) [4×5]

1. Find the projection of the line segment joining the points (3, 3, 5) and (5, 4, 3) on the straight line joining the points (2, –1, 4) and (0, 1, 5). [5]

2. A straight line makes angles $\alpha, \beta, \gamma, \delta$ with four diagonals of a cube. Prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$. Also prove that $\sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta=\frac{8}{3}$. [3+2]

3. Determine the signed distance of the point $(-2,3,-4)$ from the plane $3x+2y-6z+2=0$. [5]

4. Find the shortest distance between the straight lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$. Find also the equation of the line of shortest distance. [3+2]

5. Find the point on the sphere $x^2+y^2+z^2-2x+4y-6z-11=0$ which is nearest to the plane $3x-4z+19=0$ and determine the distance of the point from the plane. [4+1]

6. Find the equation of the right circular cone whose vertex is the origin, axis is the z-axis and which passes through the point $(3,-4,6)$. [5]

Group – B

(*Answer any four questions*) [4×10]

7. a) Show that a basic feasible solution to a linear programming problem corresponds to an extreme point of the convex set of feasible solutions. [7]

   b) If x_1 and x_2 be real, show that $X=\{(x_1,x_2)\mid x_1\le 5, x_2\ge 3\}$ is a convex set. [3]

8. a) Reduce the feasible solution $x_1=2, x_2=1, x_3=1$ of the system of equations

   $x_1+4x_2-x_3=5$

   $2x_1+3x_2+x_3=8$

   to a basic feasible solution. [5]

   b) Find the dual problem of the following linear programming problem :

   Maximize $z=6x_1+5x_2+10x_3$

   subject to $4x_1+5x_2+7x_3\le 5$

   $3x_1+7x_3\le 10$

   $2x_1+x_2+8x_3=20$

   $2x_2+9x_3\ge 5$

   $x_j\ge 0$, $j=1,3$ and x_2 is unrestricted in sign. [5]

9. a) Show that the dual of a dual is primal. [5]

b) Show that the feasible solution $x_1 = 1,\ x_2 = 1,\ x_3 = 0$ and $x_4 = 2$ to the system

$x_1 + x_2 + x_3 = 2$

$x_1 + x_2 - 3x_3 = 2$

$2x_1 + 4x_2 + 3x_3 - x_4 = 4$

$x_1, x_2, x_3, x_4 \geq 0$

is not basic. [5]

10. a) Solve the following problem by simplex method :

Maximize : $z = 2x_1 + 3x_2$

Subject to : $x_1 + x_2 \leq 8$

$x_1 + 2x_2 = 5$

$2x_1 + x_2 \leq 8$

$x_1 \geq 0,\ x_2 \geq 0$. [6]

b) Define extreme point of a convex set. State when an L.P.P. is said to be in the Canonical form. [2+2]

11. a) Determine an optimal basic feasible solution to the following transportation problem : [7]

	D_1	D_2	D_3	D_4	
O_1	2	3	11	7	6
O_2	1	0	6	1	1
O_3	5	8	15	9	10
	7	5	3	2	

b) Find an initial basic feasible solution of the following transportation problem by North-West-Corner rule : [3]

	D_1	D_2	D_3	
O_1	2	7	4	5
O_2	3	3	1	8
O_3	5	4	7	7
O_4	1	6	2	14
	7	9	18	

12. a) Find an optimal assignment to find the minimum cost for the assignment problem with the following cost matrix. [6]

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

b) Solve the L.P.P graphically

Minimize $z = 3x_1 + x_2$

Subject to : $2x_1 + 3x_2 \leq 6$

$x_1 + x_2 \geq 1$

$x_1 \geq 0$ and $x_2 \geq 0$ [4]

13. a) Solve the following L.P.P by Simplex method :

Maximize $z = 60x_1 + 50x_2$

subject to : $x_1 + 2x_2 \le 40$

$3x_1 + 2x_2 \le 60$

and $x_1, x_2 \ge 0$ [6]

b) Find the extreme points of the convex set of the feasible solutions of the following L.P.P. :

Minimize $z = 2x_1 + 3x_2 + 4x_3 + 5x_4$

subject to $2x_1 + 3x_2 + 5x_3 + 6x_4 = 16$

$x_1 + 2x_2 + 2x_3 + 3x_4 = 9$

$x_1, x_2, x_3, x_4 \ge 0$ [4]

Group – C

(*Answer any three questions*) [3×5]

14. Define the terms "absolute error", "relative error" and "percentage error" as used in numerical analysis. The number $\frac{1}{3}$ is approximated as 0·33. Find the absolute and percentage error in this approximation. [3+2]

15. Calculate f(21) from the following table

x	0	5	10	15	20
f(x)	1·0	1·6	3·8	8·2	15·4

Correct upto two decimal places. [5]

16. Find the value of f(2·5) by suitable interpolation formula from the following table (correct upto 3 significant figures) : [5]

x	2	3	4	5
f(x)	14·5	16·3	17·5	18·0

17. Find the location of the positive roots of $x^3 - 9x + 1 = 0$ and evaluate the smallest one by bisection method, correct upto 2 decimal places. [5]

18. Evaluate $\int_0^1 \frac{x}{1+x} dx$ correct upto 2 significant figures, taking six equal sub-intervals by Simpson's one-third rule. Compute the exact value of integral (correct upto 2 significant figures) and find the absolute error. [4+1]

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